

**WELCOME CITIZENS OF SCIOLY, to my Astro notes.** I apologize for their handwritten-ness. I find that writing notes out makes them a lot easier to remember than simply copying-and-pasting them from Wikipedia. But hey, maybe the illegibility of my handwriting will encourage you to re-write them!

Also... I really hate reading through pages of tiny text looking for a specific sentence, especially when the question deals with a process(star formation, for example). I much prefer looking at a visual, so my binder-making process consists mainly of condensing several articles into one (mostly) easy-to-read illustration. I get the feeling that it isn't a common technique for astronomy notetaking, so I included a lot my notes(especially ones on stellar evolution) that are in drawing form.

I also included a few comments on DSO's and my formula collection for the math section. The last thing in these notes is a list of definitions. The ones where the question asks "what is the name of the empirical function that relates the velocity dispersion and luminosity of a galaxy?" And then it gives you five options, none of which you've heard of... I started making this list last year and thought I'd share it.

Anyways, enough of my rambling, here's my notes:

## **Contents:**

### **Stellar Evolution**

- Overview
- Star/Planet Formation
- Late evolution--Low Mass
- Late evolution--Intermediate Mass
- Late evolution--High Mass
- Stellar Structure
- Degeneracy
- Masses and limits

### **Supernovae**

- SN classification(by spectra and light curves)
- Type Ia Scenarios
- Type Ia Application

### **DSO's**

- Organization strategy

### **Math**

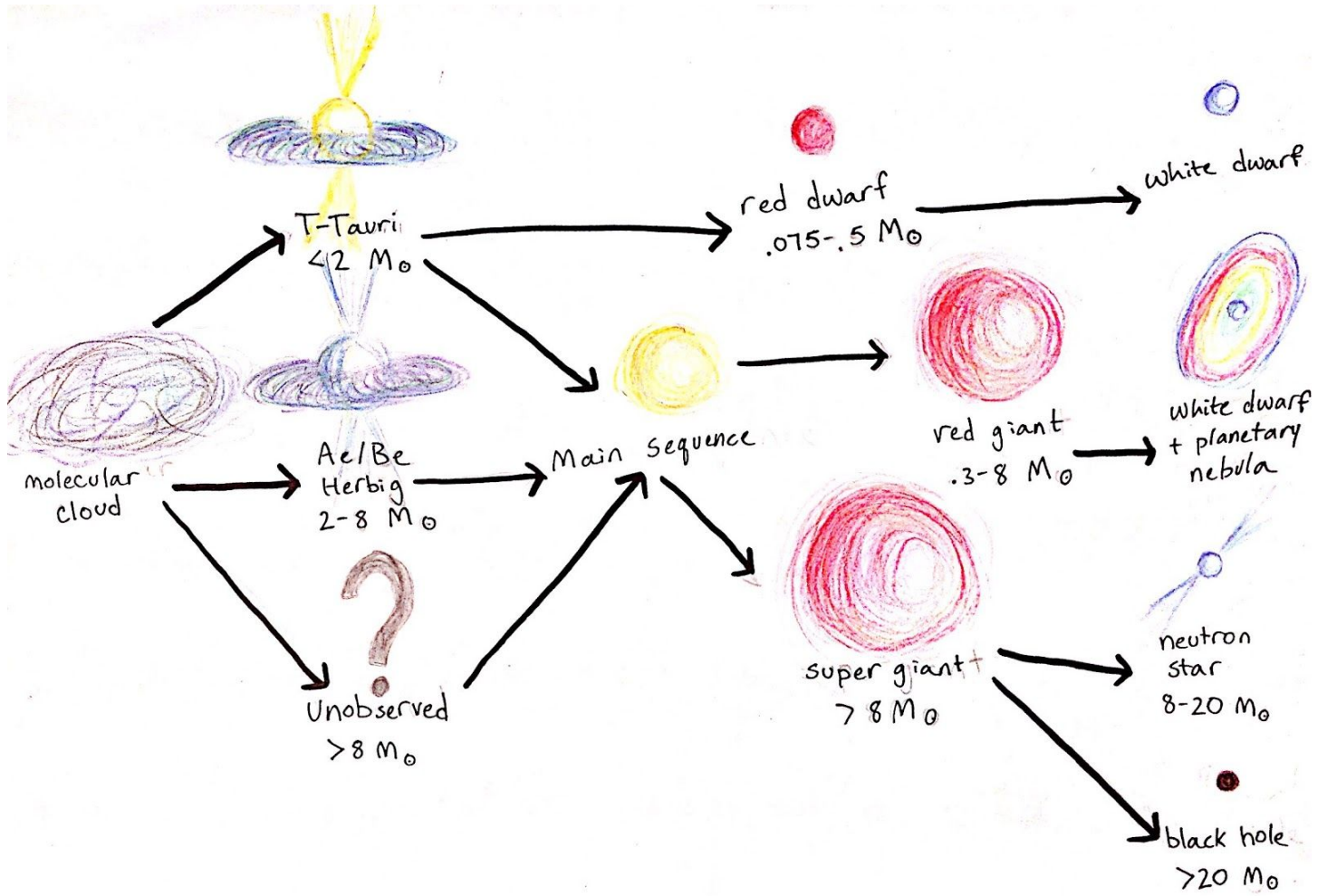
- Constants/Formulas/Derivations
- Why can't we go the speed of light?

### **Notes**

- Definition
- Star types

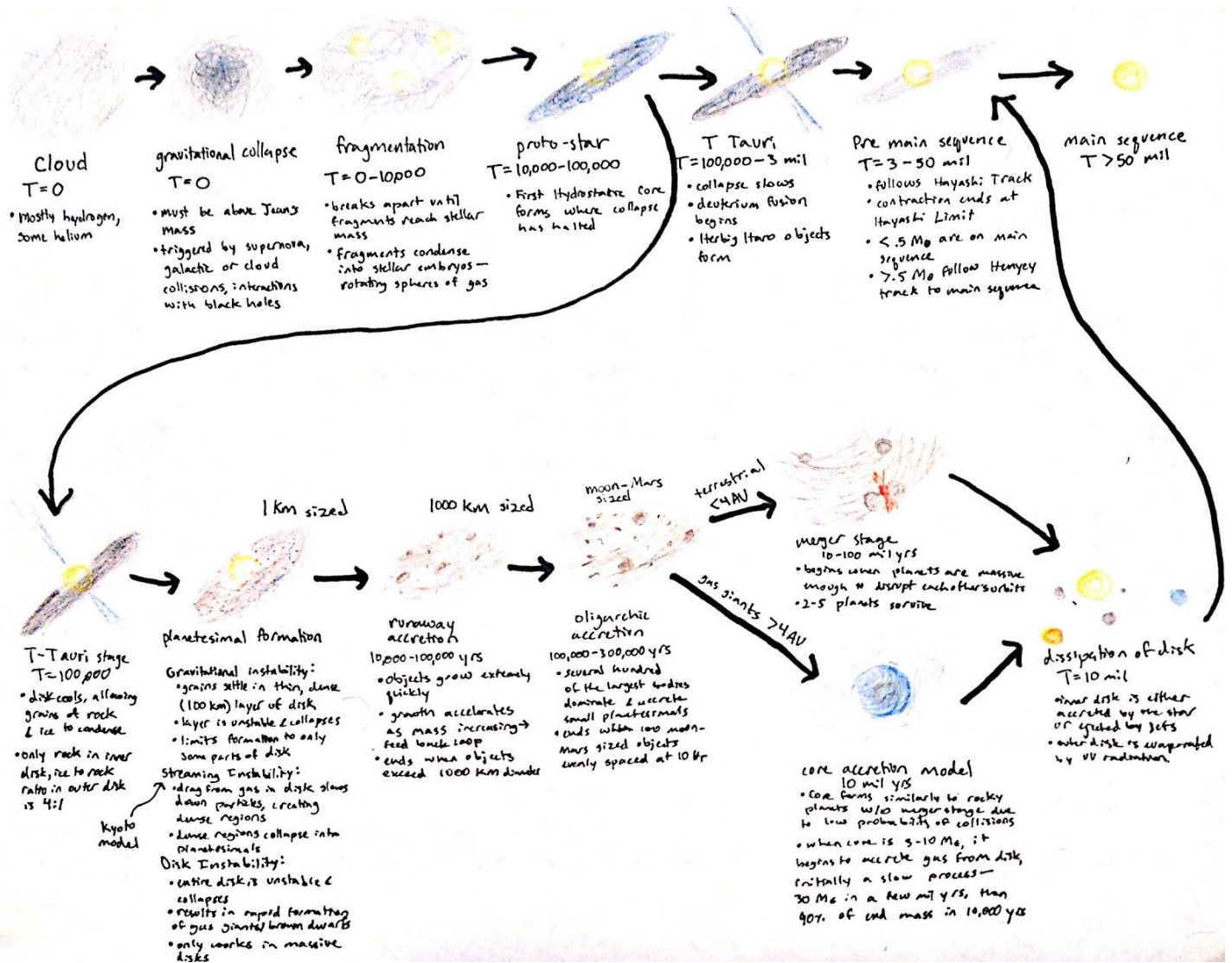
# Stellar Evolution

## Overview



# Star/planet formation

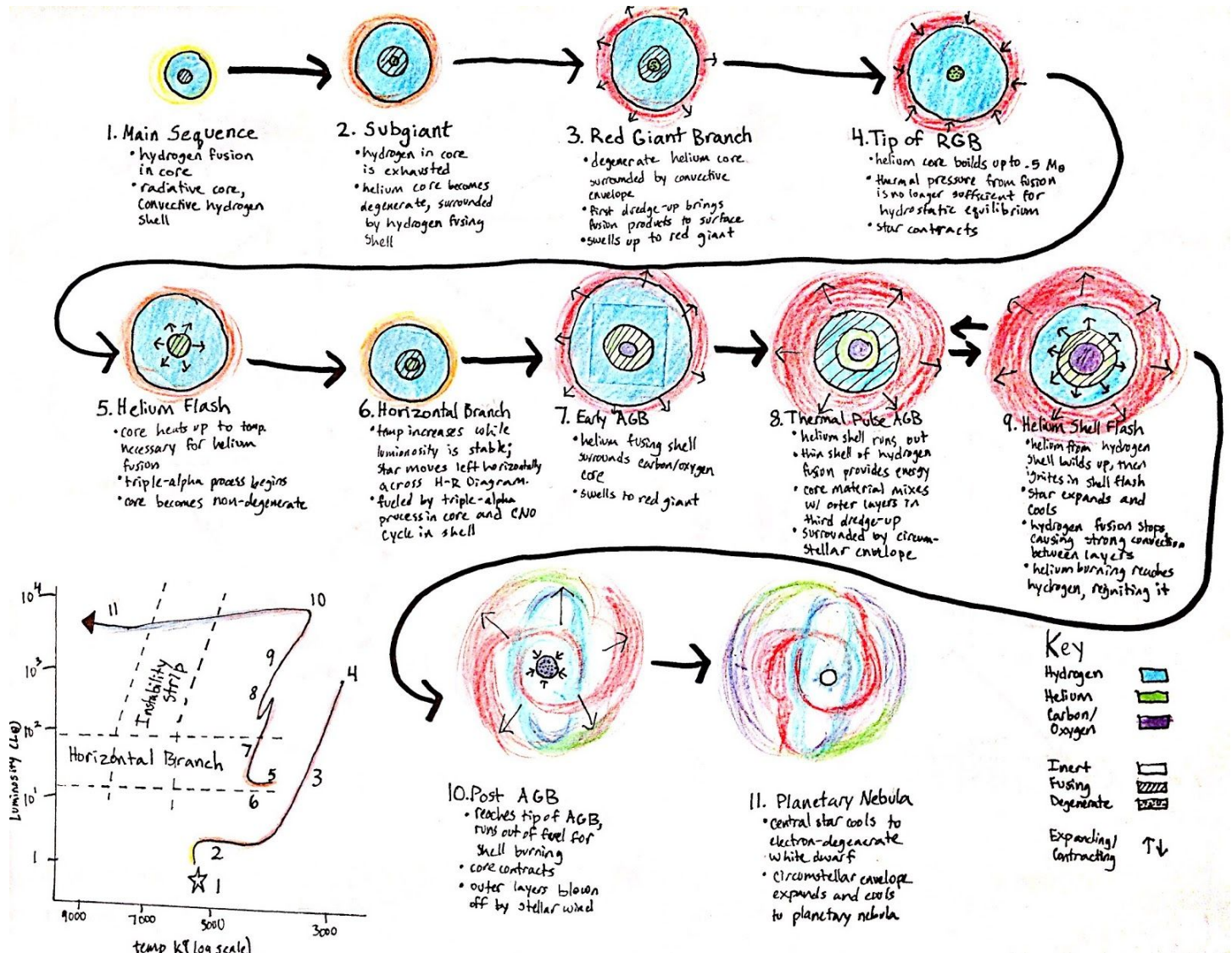
This diagram shows the formation of a relatively low mass star (<2 solar masses). 2-8 solar masses stars become Ae/Be Herbig star instead of T-Tauris, and the process is largely unknown for even more massive stars. The process of star formation is likely to be very similar regardless of mass, though the timing would vary greatly.



## Late evolution--Low Mass

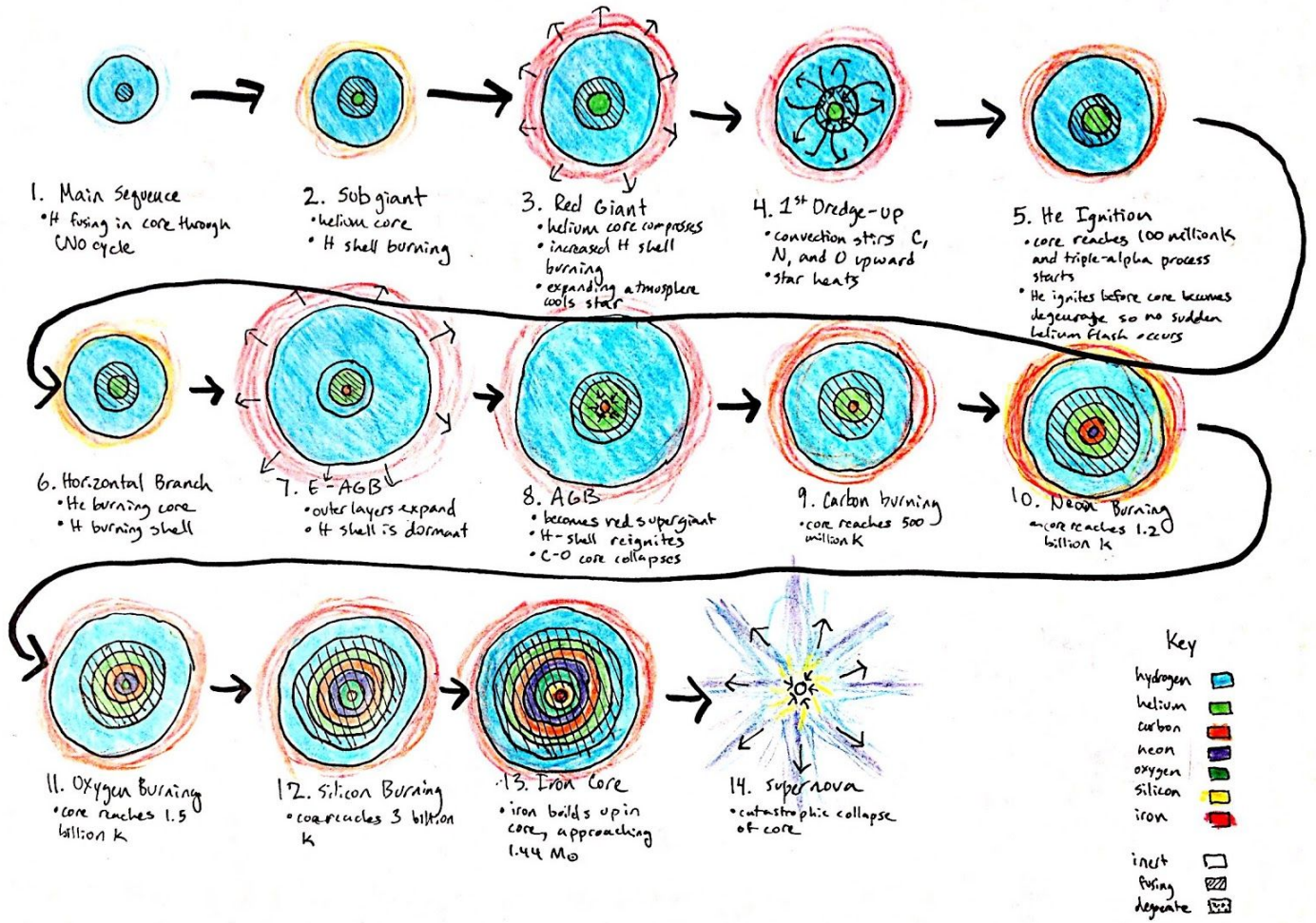
Because very low mass stars (usually red dwarfs) are fully convective, fused helium is distributed throughout the entire star, instead of piling up in the core. A degenerate helium core is never created, so the star never becomes a red giant. Instead the star directly becomes a helium white dwarf once all the hydrogen fuel has been exhausted. Because of the trillion-year lifespan of these stars, this process has not been observed.

## Late evolution-- Intermediate Mass





# Late evolution--High Mass



# Stellar Structure

low-mass

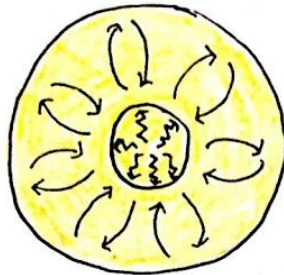
$< 0.5 M_{\odot}$



• fully convective

mid-size

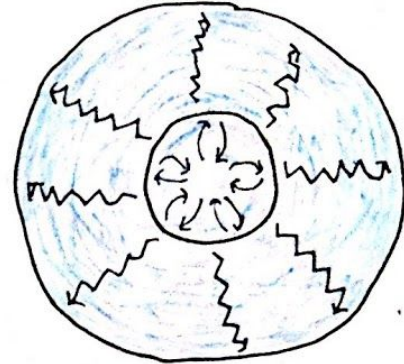
$.5 - 1.5 M_{\odot}$



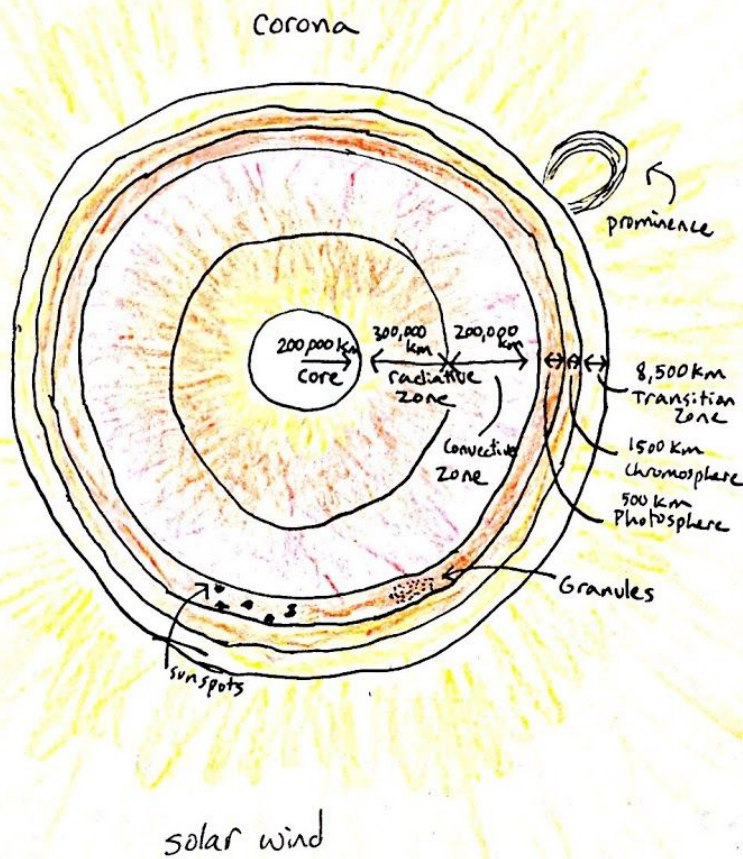
• not enough temperature gradient for convection in core

high-mass

$> 1.5 M_{\odot}$



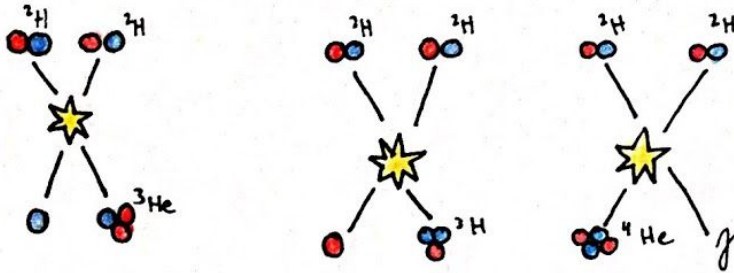
• convective core  
• hydrogen shell is fully ionized; star remains transparent to UV radiation



\* based on Sun, structure varies for higher and lower mass stars

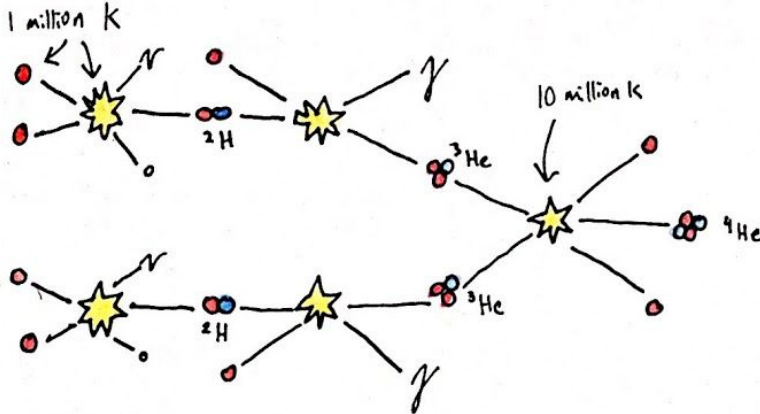
# Nuclear Processes

Deuterium:  $7.13 M_{\odot}$

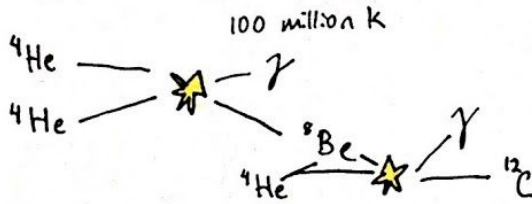


- proton
- neutron
- positron
- ν neutrino
- γ gamma radiation

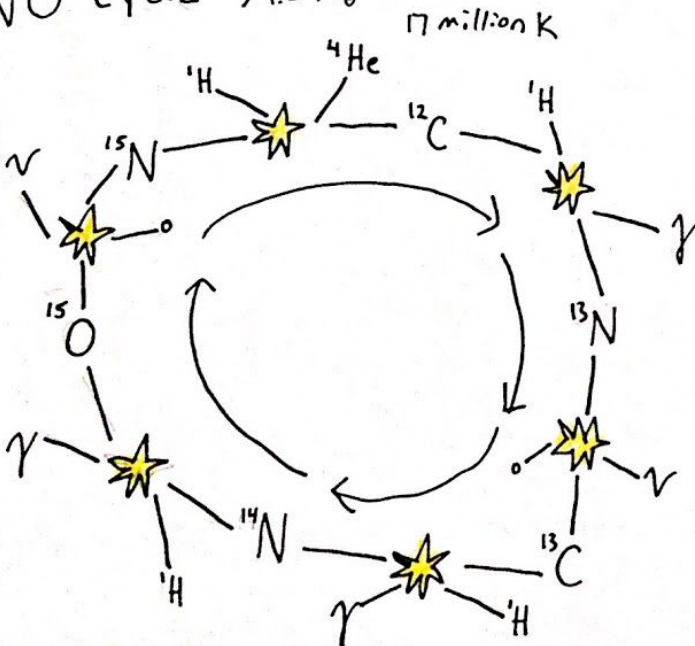
Proton-Proton:  $0.3 - 1.5 M_{\odot}$



Triple Alpha:  $> 4 M_{\odot}$



CNO cycle:  $> 1.5 M_{\odot}$





# Degeneracy

\* The object in the graphic is assumed to be electron-degenerate

**Ideal Gas Law:**  
relates the pressure, volume, and temperature of a molar amount of gas.

$$PV = nRT$$

$P$  = pressure of gas

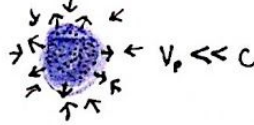
$V$  = volume of gas

$n$  = moles of gas

$R = 8.314 \frac{J}{K \cdot mol}$

$T$  = temperature

$$M = 1 M_{\odot}$$



- a degenerate gas does not obey the Ideal Gas Law; its density is so high it resists pressure
- this degeneracy pressure counters gravity and prevents collapse
- particles must move to avoid being in the same quantum state as per the Pauli Exclusion Principle

**Pauli Exclusion Principle:**



Two fermions (protons, neutrons, electrons, quarks, etc.) cannot be in the same quantum state (velocity, position, spin, charge, energy level, etc.) at the same time.

$$M = 1.2 M_{\odot}$$



- matter accumulates onto degenerate gas
- particles are squeezed into smaller and smaller spaces
- particles must move even faster to avoid being in the same quantum state

**Limits:**

- The Chandrasekhar Limit is applicable in any electron-degenerate object, including white dwarfs and the degenerate cores of massive stars
- It is analogous to the Tolman-Oppenheimer-Volkoff limit, the upper limit for any neutron degenerate object

$$M = 1.3 M_{\odot}$$



- As the mass of the degenerate gas approaches  $1.44 M_{\odot}$  (Chandrasekhar Limit), the particles' speed begins to approach the speed of light

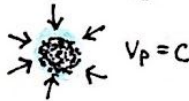
**Types of Degenerate Matter:**

- Electron (White dwarf, etc.)
- Proton (insignificant compared to electron degeneracy pressure in matter with equal amounts of protons and electrons)
- Neutrons (neutron stars)
- Quark (largely unknown)
- Preon (purely hypothetical)

And when density is so high, no degeneracy can counter gravity:

- Singularity (black hole)

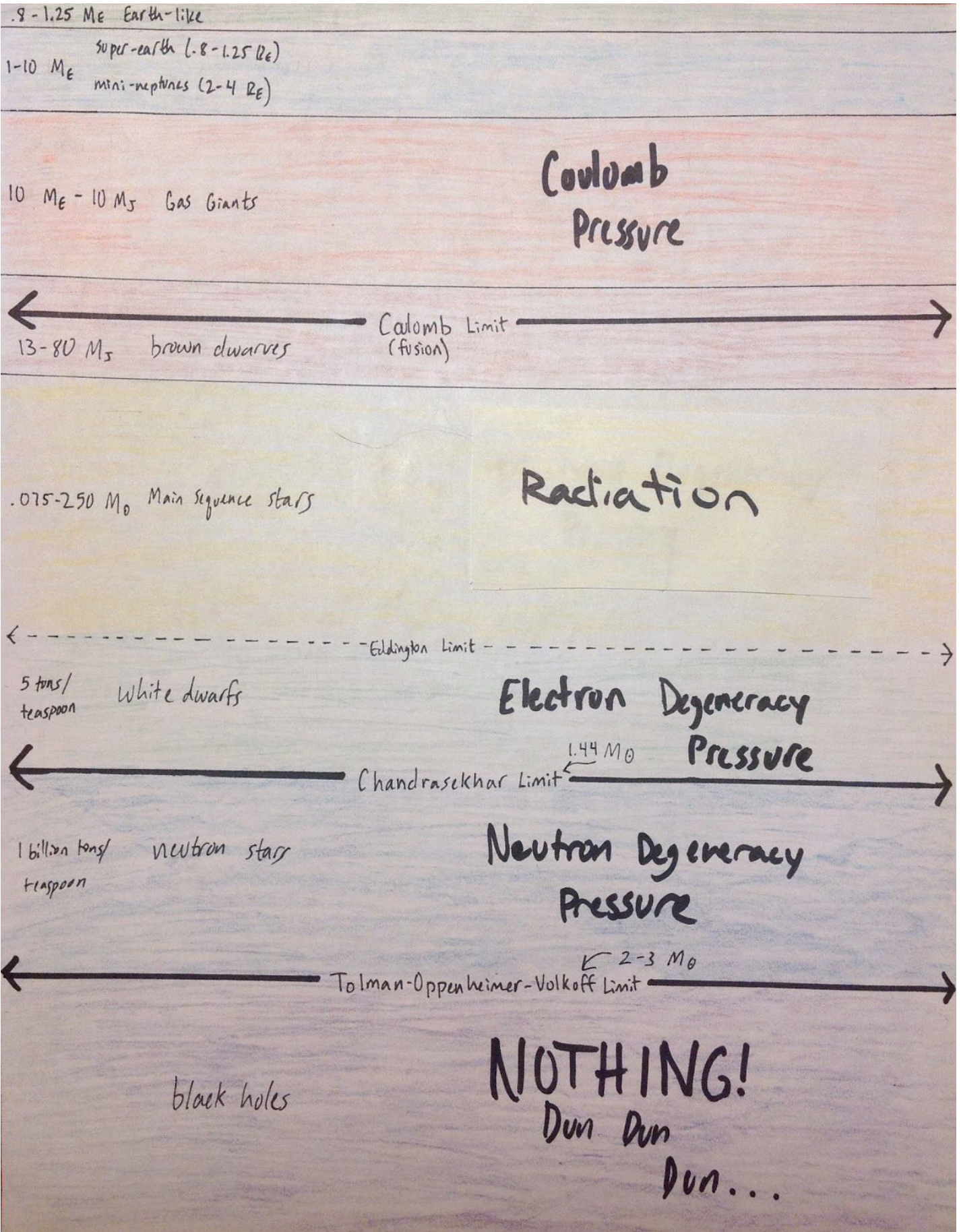
$$M = 1.44 M_{\odot}$$



- particles with mass cannot travel at the speed of light
- electron degeneracy pressure is no longer sufficient to support the object and gravity causes it to collapse

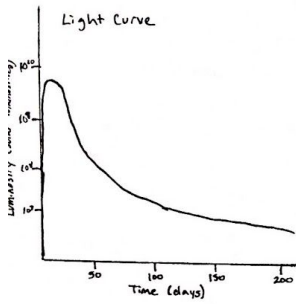


# Masses and limits



# Supernovae

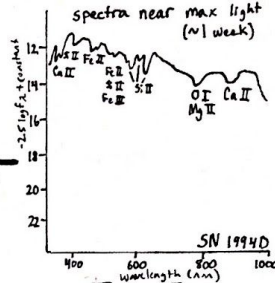
## SN Classification



Type I  
no hydrogen

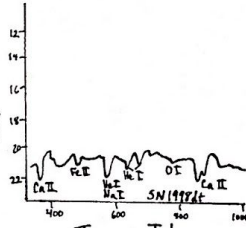
Classification System  
created by Rudolph Minkowski  
and Fritz Zwicky in 1941.

singly ionized silicon (Si II) at 615 nm



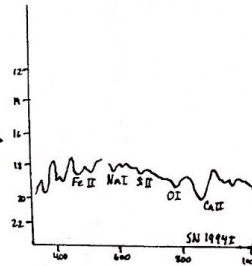
Type Ia

ionized helium (He I) at 587.6 nm



Type Ib

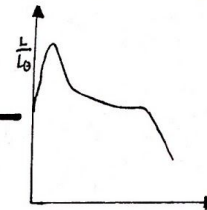
weak or no helium



Type Ic

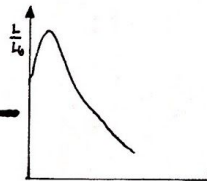
Type Ia is caused by thermonuclear runaway.  
Type Ib, Ic, and II are caused by the  
core collapse of a giant star.

reaches a plateau  
during decline

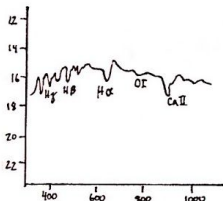


Type II-P

Linear decline



Type II-L



Type II  
hydrogen

Intermediate or  
narrow H emission  
lines in spectrum

Type II n

shows weak hydrogen  
lines initially that later  
become undetectable;  
second peaks spectrum  
resembles Type Ib

Type II b

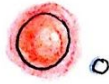


# Type Ia Scenarios

## Single Degenerate



- Main Sequence Binary**
- system with 2 MS stars
  - one is more massive and evolves quicker



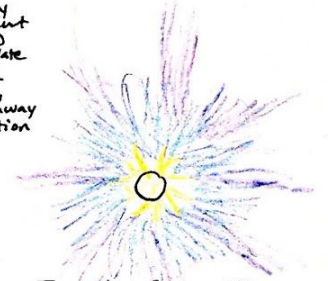
- Evolved Binary**
- more massive star leaves main sequence first, and becomes white dwarf
  - stars share a circumstellar envelope; mass loss decreases angular momentum and orbit shrinks
  - 2nd star becomes red giant



- Accretion**
- White dwarf accretes matter from red giant
  - the mass of the white dwarf approaches the Chandrasekhar limit



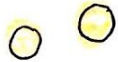
- Carbon Ignition**
- increase in mass raises temp. to 500 million K, hot enough for carbon fusion
  - Because degeneracy pressure is independent of temp., the WD is unable to regulate fusion the way a MS star would, resulting in a runaway thermonuclear reaction



## Type Ia Supernova

- $1-2 \times 10^{44}$  J released, enough to unbind star
- matter ejected at 6% the speed of light
- absolute magnitude of  $-19.3$

## Double Degenerate



- Main Sequence Binary**
- system with 2 MS stars
  - similar in mass and evolve at same rate



- White Dwarf Binary**
- stars evolve into white dwarfs



- Orbit Decay**
- orbit decays over time due to gravitational radiation (gravitational waves reduce angular momentum)
  - orbit becomes extremely small



- Merge**
- stars merge with each other
  - combined mass exceeds Chandrasekhar Mass
  - electron degeneracy is insufficient to support star

## Supernova Application

Because all Type Ia supernova explode when the white dwarf reaches 1.44 solar masses, they all have the same energy output, therefore the same absolute magnitude. This makes them great standard candles; astronomers use them to measure intergalactic distances, when parallax and Cepheid variables no longer work. Distance calculations for supernovae provided the first direct evidence of dark energy. Below are some example of finding distance with Ia supernovae.



- 1) a. You observe a Type Ia supernova coming from the direction of the Andromeda Galaxy. It has an apparent magnitude of 5.16. How many light years is it from Earth?

$$\text{distance modulus} \rightarrow m - M = 5 \log(d) - 5$$

$$d = 10^{(m - M + 5)/5}$$

$$\text{absolute mag of Ia SN} \rightarrow \begin{aligned} m &= 5.16 \\ M &= -19.3 \end{aligned}$$

$$d = 10^{(5.16 - (-19.3) + 5)/5}$$

$$d = 10^{5.892}$$

$$d = 779830.11 \text{ pc}$$

$$d = 2.54 \times 10^6 \text{ ly}$$

- b. Is this supernova the right distance from Earth to have originated from the Andromeda Galaxy?

$$d_{\text{Andromeda}} = 2.537 \times 10^6 \text{ ly}$$

$$d_{\text{measured}} = 2.54 \times 10^6 \text{ ly}$$

Yes

- 2) a. A Type Ia supernova is observed in a galaxy we know to be 100 million light years away. What is the apparent magnitude of the supernova?

$$\text{distance modulus} \rightarrow m - M = 5 \log(d) - 5$$

$$\text{absolute mag of Ia SN} \rightarrow \begin{aligned} d &= 10^8 \text{ ly}, 3.07 \times 10^7 \text{ pc} \\ M &= -19.3 \end{aligned}$$

$$m - (-19.3) = 5 \log(3.07 \times 10^8) - 5$$

$$m - (-19.3) = 5(7.49) - 5$$

$$m - (-19.3) = 32.44$$

$$m = 13.14$$

- b. Would this supernova be visible with the naked eye?

$$m_{\text{visible}} = 6$$

$$m_{\text{observed}} = 13.14$$

NO







# DSO's

## Organization strategy

I haven't started working on DSO notes yet, and I doubt I will until this year's are announced. However, I just wanted to share my organization strategy as I found it to be very effective. I organized all the DSO's on a chart, along with their discoverer, discovery method, distance to Earth, age, composition... pretty much every detail I could find. Also, I had a column for both pictures and graphs (although really I should have had one for artist's impressions and actual data).

I found this chart to be extremely effective because I *hate* leafing through pages of text looking for the right line. This way, when I was asked "what exoplanet was discovered in 1995, and what was its significance?" I just had to go down the discovery date column until I found 1995. Oh that's 51 Pegasi b, and looking at the significance column, it was the first planet discovered around a sunlike star.

Below I have half the chart so you can see what kind of stuff I put on it and decide whether you like it. It is sideways though, but I figured you won't actually want to use the info from last year on it.

Object	Star	planet(s)	detection	discovery	pictures/graphs	distance	significance
51 Pegasi b	M5 G-type 5 <sup>th</sup> mag	Hot Jupiter	• radial velocity	Oct 6, 1995 Michael Mayor, Didier Queloz ELODIE spectrograph in France		51 ly	• first planet discovered around a MS(sun-like) star
HD 95086	Pre-MS F-type 8 <sup>th</sup> mag	gas giant	• direct imaging	Dec 4, 2013 Vanessa Bailey Magellan Telescopes		300 ly	• oldest 73 much larger than transit possible, questions about formation • 1/100th found to be around a star like our sun
WASP 43b	young K-type 12 <sup>th</sup> mag	Hot Jupiter	• transit • radial velocity • reflection/emission modulations	April 15, 2011 CoRoT Helix, La Silla observatory		260 ly	• shortest orbital period when discovered • weather patterns
WASP 18b	F-type 9 <sup>th</sup> mag	Hot Jupiter (merging w/ star)	• transit • radial velocity	Aug 21, 2009 CoRoT Helix, SuperWASP		325 ly	• expected to merge w/ star due to tidal deceleration • planet is absorbing stars magnetic field
HD 95086	Pre-MS A-type 7 <sup>th</sup> mag	gas giant	• direct imaging in infrared	June 25, 2013 "Very large" telescope		246 ly	• younger analog of HD 95086
HR 8199	F-type-I K-line suggests A5 6 <sup>th</sup> mag	gas giant	• direct imaging	Nov. 13, 2008 Christian Marois, Hubble		129 ly	• first where orbital motion was confirmed by direct imaging



radius	mass	temp	distance	orbit	habitability	age	composition	disk
	5-37 $M_J$		43-302 AU		no	1-300 mya		habitability may include forming planet
1.9 $R_J$	.5 $M_J$ 150 $M_E$	1265 K	.05 AU	4 days	yes	6-8 bya	*silicate clouds?	
.42 $R_J$ 4.59 $R_E$	.08 $M_J$ 26 $M_E$	890 K	.05 AU	4.9 days		6.5 bya	*H <sub>2</sub> O vapor *redishish cloud free *90% heavy elements	
1 $R_J$	2 $M_J$	1710 K	.015 AU	.8 day	yes	>100 mya	moisture in prot	
1 $R_J$	10 $M_J$	2000 K	.02 AU	.9 day	yes	500 mya - 2 bya		
71 $R_J$ (61:94:114)	5 $M_J$	1000 K	56-61 AU		no	17 mya		*2nd system w/ large elements *6 clear disk *other disk & hot inner disk, cool outer
	11 $M_J$	1800 K	650 AU		no	13 mya		*debris disk
	7 $M_J$ 10 $M_J$ 10 $M_J$ 9 $M_J$			460 years 190 years 100 years 46 years	no	30 mya	*carbon dioxide (CO <sub>2</sub> ) *methane (CH <sub>4</sub> ) *Ammonia (NH <sub>3</sub> ) *highly low (C <sub>2</sub> H <sub>6</sub> )	*debris disk *excess IR emission

# Math

## Formulas/Derivations

I had a very hard time finding a good list of formulas last year (I did, however, find one full of typos so look out for that one). Here's all the formulas I could find, along with derivations (for Kepler's Laws and other orbital mechanics). They're handwritten though (adding symbols is a nightmare) so no copying-and-pasting. Unfortunately, you will either have to rewrite or type them.



# CONSTANTS:

## Theory:

speed of light  $c = 3.0 \times 10^8 \frac{m}{s}$

gravitational constant  $G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$

Hubble Constant  $H_0 = 70 \frac{m/s}{Mpc}$

Planck Constant  $h = 6.63 \times 10^{-34} J \cdot s$

Coulomb's Constant  $k = 1.38 \times 10^{-23} \frac{J}{K}$

Stefan-Boltzmann Constant  $\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 K^4}$

## Solar system:

solar luminosity  $L_0 = 3.84 \times 10^{26} W$

solar mass  $M_0 = 1.99 \times 10^{30} kg$

solar radius  $R_0 = 6.96 \times 10^8 m$

solar temperature  $T_0 = 5800 K$

Earth mass  $M_E = 5.97 \times 10^{24} kg$

Earth radius  $R_E = 6.37 \times 10^6 m$

Jupiter mass  $M_J = 1.89 \times 10^{27} kg$

Jupiter radius  $R_J = 7.15 \times 10^7 m$

## Distance:

mega parsec  $Mpc = 1,000,000 pc$

parsec  $pc = 3.26 ly$   
 $206,265 AU$

light year  $ly = .307 pc$   
 $9.46 \times 10^{15} m$

Astronomical Unit  $AU = 1.49 \times 10^{11} m$

micrometer  $\mu m = 10^{-6} m$

nanometer  $nm = 10^{-9} m$

Ångström  $\text{Å} = 10^{-10} m$

## DISTANCE/MAGNITUDE:

parallax  $d = \frac{1}{p}$   $d = \text{distance in pcs}$   
 $p = \text{parallax angle in arc seconds}$

distance modulus  $m - M = 5 \log d - 5$   $m = \text{apparent magnitude}$   
 $M = \text{absolute magnitude}$   
 $d = 10^{(m-M+5)/5}$   $d = \text{distance in pcs}$

small angle formula  $D = \frac{\alpha d}{206,265}$   $D = \text{linear diameter in AU}$   
 $\alpha = \text{angular size in arc seconds}$   
 $d = \text{distance to object in AU}$

inverse square law  $b = \frac{L}{4\pi d^2}$   $b = \text{apparent brightness in W/m}^2$   
 $L = \text{luminosity in W}$   
 $d = \text{distance in m}$

apparent magnitude/color ratios  $m_2 - m_1 = -2.5 \log_{10} \frac{b_2}{b_1}$   $m = \text{mag of star (log 2)}$   
 $b = \text{brightness of star (log 2)}$   
 $m_b - m_v = -2.5 \log_{10} \frac{b_b}{b_v}$

## STELLAR PROPERTIES:

luminosity  $L = 4\pi R^2 \sigma T^4$   $L = \text{luminosity in W}$   
 $R = \text{radius in m}$   
 $T = \text{temp in K}$

main sequence life expectancy  $T_{\text{ms}} = M^{-2.5} \cdot T_0$   $T = \text{life in Gyr}$  \* exponent varies by mass  
 $M = \text{mass in } M_{\odot}$

luminosity ratio  $\frac{L}{L_{\odot}} = \left(\frac{R}{R_{\odot}}\right)^2 \cdot \left(\frac{T}{T_{\odot}}\right)^4$   $R = \text{radius in m}$   
 $T = \text{temp in K}$

Jean's mass  $\frac{5B_c kT}{Gm} = M_J$   $R_c = \text{radius of cloud (m)}$   
 $T = \text{temperature in K}$   
 $m =$   
 $M_J = \text{Jean's mass in kg}$



# STELLAR VELOCITIES:

Radial Velocity

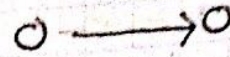


$V_r$  is measured from doppler shift:

$$V_r = \frac{\Delta\lambda}{\lambda} \cdot c$$

$V_r$  = radial velocity in m/s  
 $\Delta\lambda$  = wavelength shift  
 $\lambda$  = rest wavelength

Tangential Velocity

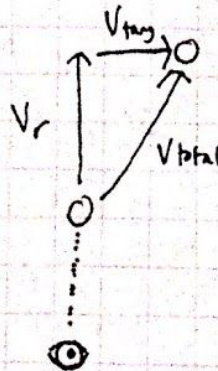


$V_{tang}$  is measured from proper motion:

$$V_{tang} = \frac{4.75 \cdot \mu}{p}$$

$V_{tang}$  = tangential velocity in km/s  
 $\mu$  = proper motion in arc sec/yr  
 $p$  = parallax in arc sec

Total Velocity



$$V_{total} = \sqrt{V_r^2 + V_{tang}^2}$$

\*be sure  $V_r$  &  $V_{tang}$  are in the same units



**Planet-related questions** will likely not be on this year's test. However, many of these formulas describe binary systems in general; because Type Ia supernovae result from binary systems, they might be applicable anyway.

## PLANET PROPERTIES:

surface gravity

$$g = \frac{GM}{r^2}$$

$g$  = gravitational acceleration  $m/s^2$

$M$  = mass in kg

$r$  = radius in m

equilibrium temp

$$T_{eq} = \left( \frac{L(1-a)}{16\sigma\pi D^2} \right)^{1/4}$$

$L$  = luminosity of star in W

$a$  = albedo

$D$  = distance from star in m

$T_{eq}$  = temp in K

$$\text{energy received: } 4\pi R_0^2 \sigma T_0^4 \times \left( \frac{R_p}{2d} \right)^2$$

$$\text{energy emitted: } 4\pi R_p^2 \sigma T_p^4$$

escape velocity

$$V = \sqrt{\frac{2GM}{r}}$$

$V$  = escape velocity in m/s

$m$  = mass in kg

$r$  = radius in m

Newton's law of gravitation

$$F = G \frac{m_1 m_2}{r^2}$$

$F$  = force in N

$m$  = mass in kg

$r$  = distance in m

tidal forces

$$F = \frac{2GMmd}{r^3}$$

$F$  = force in N

$M$  = mass in kg

$d$  = distance between objects

$r$  = radius of affected object

radial velocity of star

$$r_s = \frac{r_{total}}{(m_s/m_p) + 1}$$

$r_s$  = star to barycenter in m

$r_{total}$  = distance between star & planet in m

$m_s$  = mass of star in kg

$m_p$  = mass of planet in kg

$v_s$  = velocity of star in m/s

$p$  = orbital period in s

$$v_s = \frac{2\pi r_s}{p}$$

$v$  = actual radial velocity

$v_{observed}$  = observed velocity

$\theta$  = angle of inclination

$$\text{inclined orbit} \rightarrow v_{observed} = v \sin \theta$$

binary systems

$$\frac{m_a}{m_b} = \frac{r_b}{r_a}$$

$m$  = mass

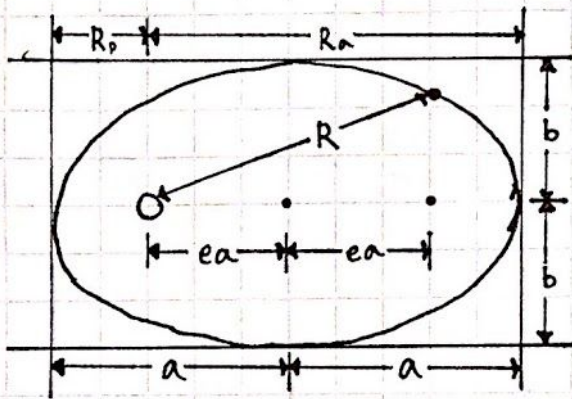
$r$  = distance to barycenter

$v$  = orbital velocity

$$\frac{v_a}{v_b} = \frac{r_b}{r_a}$$

$$\frac{m_a}{m_b} = \frac{v_b}{v_a}$$

Kepler's first law: planets orbit in ellipses, with the star at one focus



$a$  = semi major axis

$b$  = semi minor axis

$e$  = eccentricity

$R$  = distance from star to planet

$R_p$  = distance at perihelion

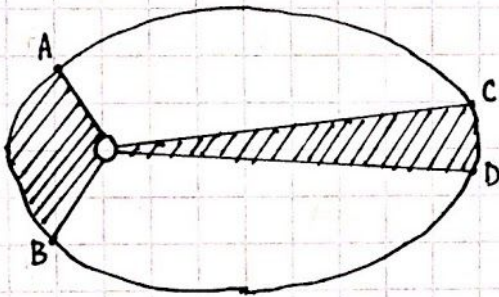
$R_a$  = distance at aphelion

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$R_p = a(1 - e)$$

$$R_a = a(1 + e)$$

Kepler's second law: planets carve out equal areas in equal times



$$dA = \frac{1}{2} r \cdot r d\theta \leftarrow \text{area of triangle}$$

$$\frac{dA}{dt} = \frac{1}{2} r \cdot r \frac{d\theta}{dt}$$

written as  
← a rate

$$= \frac{1}{2} r v_\theta$$

$$\vec{L} = m(\vec{r} \cdot \vec{v}) \leftarrow \text{definition of}$$

$$L = m r v_\theta \leftarrow \text{angular momentum}$$

$$\frac{dA}{dt} = \frac{L}{2m} \leftarrow \text{combine the two equations}$$

Kepler's third law: the square of the period of an orbit is proportional to the cube of the semi-major axis

$$p^2 \propto a^3$$

$$p^2 = \frac{4\pi^2}{G(M_1 + M_2)} a^3$$

$$p^2 = \frac{a^3}{M}$$

$p$  = in s

$M$  = in kg

$a$  = in m

$p$  = in yrs

$M$  = in  $M_\odot$

$a$  = in AU



## Orbital speed:

Circular Velocity:

$$r = a$$

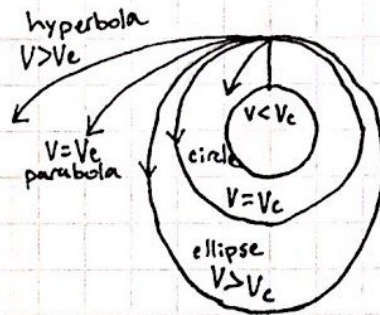
$$m \ll M$$

$$v_c = \frac{2\pi a}{P}$$

Kepler's 3<sup>rd</sup> law  $\rightarrow P^2 = \frac{4\pi^2 a^3}{GM}$

$$\frac{2\pi a}{v_c} = \sqrt{\frac{4\pi^2 a^3}{GM}}$$

$$v_c = \sqrt{\frac{GM}{a}}$$



$$v_c = \sqrt{\frac{GM}{r}}$$

$$v_e = \sqrt{\frac{2GM}{r}}$$

Elliptical Orbits:

$$E = U + K$$

E = total energy

U = potential energy

K = kinetic energy

$$E_{\text{total}} = -\frac{GMm}{a} + \frac{1}{2}mv^2$$

$$E_{\text{total}} = -\frac{GMm}{a} + \frac{1}{2}\frac{GMm}{a}$$

$$E_{\text{total}} = -\frac{GMm}{2a}$$

$$E = U + K$$

$$-\frac{GMm}{2a} = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$

vis viva  $\rightarrow v^2 = G(M+m)\left(\frac{2}{r} - \frac{1}{a}\right)$

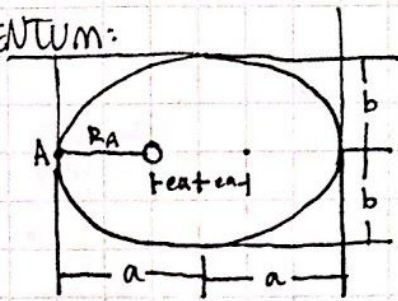


## CONSERVATION OF ANGULAR MOMENTUM:

$$\text{vis viva: } v^2 = GM \left( \frac{2}{r} - \frac{1}{a} \right)$$

$$L = r \cdot p$$

$$L = r \cdot m \cdot v \cdot \sin \theta$$



$$R_A = a - ea$$

$$v^2 = GM \left( \frac{2}{a - ea} - \frac{1}{a} \right)$$

$$v^2 = GM \left( \frac{2a}{a(a - ea)} - \frac{a - ea}{a(a - ea)} \right)$$

$$v^2 = GM \left( \frac{2a - a - ea}{a(a - ea)} \right)$$

$$v^2 = \frac{GM}{a} \cdot \frac{1 + e}{1 - e}$$

$$L = r \cdot m \cdot v \cdot \sin \theta$$

$$L = m v r_A$$

$$L = m v (a - ea)$$

$$L = m (GM/a)^{1/2} (1 + e)^{1/2} (1 - e)^{-1/2} a$$

$$L = m a (GM/a)^{1/2} (1 + e)^{1/2} (1 - e)^{1/2}$$

$$L = m a (GM/a)^{1/2} [(1 + e)(1 - e)]^{1/2}$$

$$L = m a (GM/a)^{1/2} (1 - e^2)^{1/2}$$

$$L = m a (GM/a)^{1/2} \left( 1 - 1 + \frac{b^2}{a^2} \right)^{1/2}$$

$$L = m a (GM/a)^{1/2} \frac{b}{a}$$

$$L = m (GM/a)^{1/2} b$$

$$L = m v_0 b$$

$v_0$  = speed of satellite in circular orbit



## ENERGY (OF AN ORBIT):

$$E_{\text{total}} = E_{\text{kinetic}} + E_{\text{potential}}$$

$$E_{\text{total}} = \frac{1}{2} m v^2 - G(m_1 \cdot m_2) / R$$

$$\frac{m v^2}{R} = \frac{G(m_1 \cdot m_2)}{R^2}$$

$$m v^2 = \frac{G(m_1 \cdot m_2)}{R}$$

$$E_{\text{total}} = -G(m_1 \cdot m_2) / 2R$$



$$E_{\text{mech}} = -G \frac{M m}{2a}$$

$$r = r_{\text{min}}$$

$$E_{\text{potential}} = \frac{2 E_{\text{mech}}}{(1-e)}$$

$$E_{\text{kinetic}} = \frac{-E_{\text{mech}} (1+e)}{(1-e)}$$

$$r = r_{\text{max}}$$

$$E_{\text{potential}} = \frac{2 E_{\text{mech}}}{(1+e)}$$

$$E_{\text{kinetic}} = \frac{-E_{\text{mech}} (1-e)}{(1+e)}$$

Virial theorem:  $E_{\text{kinetic}} = -\frac{E_{\text{potential}}}{2}$

$$E_{\text{potential}} = -2 E_{\text{kinetic}}$$

# LIGHT:

Wien's Law:  
peak wavelength

$$\lambda = \frac{2.9 \cdot 10^6}{T}$$
$$\lambda = \frac{2.9 \cdot 10^9}{T}$$

$\lambda$  = peak wavelength in nm  
 $T$  = temp in K

$\lambda$  = peak wavelength in Angstroms  
 $T$  = temp in K

doppler shift

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

$\Delta\lambda$  = wavelength shift  
 $\lambda$  = wavelength of source  
 $v$  = velocity

Hubble's Law

$$v = H_0 d$$

$v$  = recessional velocity km/s  
 $d$  = distance Mpc

Planck-Einstein  
Relation

$$E = hv$$

$$E = \frac{hc}{\lambda}$$

$E$  = energy  
 $v$  = frequency  
 $\lambda$  = wavelength

Stefan-Boltzmann  
Law

$$F = \sigma T^4$$

$F$  = Energy flux in  $\frac{W}{m^2}$   
 $T$  = Temp in K

radial  
Velocity

$$\frac{\Delta\lambda}{\lambda} = \frac{v_r}{c}$$

$\Delta\lambda$  = wavelength shift  
 $\lambda$  = rest wavelength  
 $v_r$  = radial velocity

## Why can't we go the speed of light?

This is not directly related to astro, but I thought I'd mention it anyway. I hear this misconception all the time: "It's not that we can't go faster than the speed of light, it's just that we haven't figured out how yet." Grrr. So I've included this visual (ha! Who would've guessed that!) proof. I saw it on the minutephysics YouTube channel if you want to go watch that.



# Speed of Light:

Einstein's full equation:

$$E^2 = (mc^2)^2 + (pc)^2$$

$E$  = Energy

$m$  = Mass

$c$  = speed of light

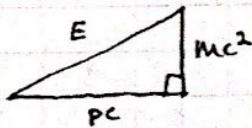
$p$  = momentum

$$E^2 = (mc^2)^2 + (pc)^2$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$c^2 = a^2 + b^2$$

Thus, Einstein's equation can be drawn as a right triangle:



If the object isn't moving and  $p=0$ ;

$$\frac{E}{mc^2}$$

$$E = mc^2$$

If the object has no mass and  $m=0$ ;

$$\frac{E}{pc}$$

$$E = pc$$

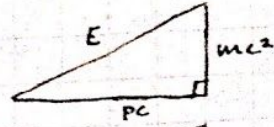
The object's velocity is the ratio of its momentum to energy, as a factor of  $c$ :

$$v = \frac{pc}{E} \cdot c$$

As mass decreases, the momentum-to-energy ratio increases, approaching 1...

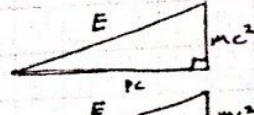
$$\frac{pc}{E} = .89$$

$$v = .89c$$



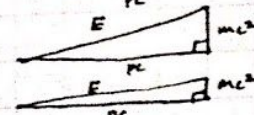
$$\frac{pc}{E} = .93$$

$$v = .93c$$



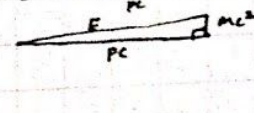
$$\frac{pc}{E} = .97$$

$$v = .97c$$



$$\frac{pc}{E} = .99$$

$$v = .99c$$



... ergo the velocity approaches  $c$ , but cannot reach it until  $m=0$ .

$$\frac{pc}{E} = 1 \quad \frac{E}{pc} \quad m=0$$

$$v = c$$

Thus, as long as the object has mass, the ratio of momentum to energy (of a right triangle's leg to its hypotenuse) will always be less than 1, and velocity can never reach  $c$ .

Spaceships (and humans) have mass, so it's impossible for them to fly at light speed.

Conversely, if an object has no mass, that ratio will always be 1, and the velocity will always be  $c$ .

Massless particles must travel at the speed of light; they cannot slow down.

Also, it's now obvious that  $c$  is not the speed of light, it's the speed of any massless particle (photon, gluon, etc.). Understanding this makes  $c$  seem less arbitrary, eliminating questions like, "why light? Why can't we build something even faster with the right technology?"

# Notes

Here's some alphabetical lists with brief descriptions/definitions that you might encounter.

## Definitions

**Ambipolar Diffusion**--the diffusion of positively and negatively charged particles via their interactions with a magnetic field.

**Baade Wesselink Method**--for calculating properties of Cepheids.

**Chandrasekhar Limit**--the amount of mass a white dwarf can have before a 1a SN(the upper limit to an electron degenerate object). Accepted to be 1.44 solar masses.

**Coulomb Barrier**--the energy barrier that 2 atomic nuclei must overcome to fuse. Governs the volume of planets.

**Degenerate Matter**--Matter at such a high density its pressure is independent of temperature.

**Eddington Limit**--the maximum luminosity an object can have while maintaining a balance between outward radiation and inward gravitational force (hydrostatic equilibrium).

**Electron Degeneracy Pressure**--results because electrons resist being too close to each other.

**Faber-Jackson Relation**--an empirical function that relates the velocity dispersion and the luminosity of a galaxy, used to determine distance.

**Gamow Peak**--the product of Maxwellian distribution and tunneling rate under which fusion is most likely to occur.

**Hawking Radiation**--theoretical electromagnetic radiation emitted by black holes.

**Hill Sphere**--also called "*Roche Sphere*" The region where an object dominates the attraction of satellites.

**Humphreys-Davidson Limit**--upper stellar luminosity limit.

**Hydrodynamic Escape**--when the atmosphere of a gas giant is stripped away by its star.

**Ideal Gas Law**--Function that relates a gas's volume, pressure, temperature, and quantity.

**Initial Mass Function**--an empirical function that describes the distribution of initial mass for a population of stars (a histogram for star populations).

**Jeans Mass**--the amount of mass needed for a dust cloud to collapse into a star.

**Kappa Mechanism**--the driving mechanism behind the changes in luminosity in pulsating variables. Greek letter Kappa is used to indicate radiative opacity.

**Kelvin Helmholtz Mechanism**--the process that occurs when the surface of a star or planet drops. The cooling causes the pressure to drop, resulting in the object shrinking. This raises the pressure in the core, heating up the object.

**Kirchhoff's Law**--A hot dense gas produces a continuous spectrum without dark lines; a hot diffuse gas produces emission lines; a cool diffuse gas produces dark absorption lines when placed in front of a continuous spectrum.

**Kozai Mechanism**--the periodic exchange of an object's orbit's inclination and eccentricity.

**Nebular Hypothesis**--the theory that our solar system formed from a nebula.

**Neutron Degeneracy Pressure**--results because neutrons resist being too close to each other.

**(O-C) Method**--"Observed minus Calculated" method, used to determine changes in the period of a variable star.

**Olber's Paradox**--If the universe is truly infinite, the entire sky should be filled with light.

**Pauli Exclusion Principle**--no two electrons in an atom can be in the same configurations at the same time. Applies to all fermions, but not to bosons.

**Poynting-Robertson Drag**--solar radiation causes dust to lose angular momentum, causing it to spiral towards the star.

**Rayleigh Scattering**--scattering of light in a medium. Causes the sky to be blue and sunsets to be red.

**Roche Limit**--the distance within which an object will disintegrate due to another object's tidal forces overcoming its own gravity.



**Roche Lobe**--the tear-drop shaped area around a binary star where the material is gravitationally bound to it.

**Roche Limit**--See "*Hill Sphere*"

**Rossiter-McLaughlin Effect**--When an exoplanet transits, the star rotates on its axis, created a small red and blue shift of its atmosphere that can be detected.

**Rydberg Formula**--gives energy differences between levels in Bohr Model.

**Schonberg-Chandrasekhar Limit**--the maximum mass of a non-fusing, isothermic core that can support an enclosing envelope.

**Schwarzschild Radius**--the radius something must be compressed to to form a black hole (for the escape velocity to become the speed of light)

**Stefan-Boltzmann Law**--describes the power radiated from a black body in terms of its temp.

**Stromgen Radius**--the size of an HII region.

**Sudarsky Classes**--a system of classifying gas giant planets.

**Tolman-Oppenheimer-Volkoff Limit**--upper mass limit for neutron stars (or any neutron degenerate object).

**Tully-Fischer Relation**--a technique that uses the rotational velocity of a galaxy to determine distance.

**Virial Theorem**--for a stable, self-gravitating, spherically distributed object (stars, galaxies, etc.) the kinetic energy must equal the potential energy within a factor of 2.

**Vogt-Russell Theorem**--the structure of a star depends only on its mass and the distribution of chemical elements in its interior.

**Wien's Law**--energy emitted by objects of different temperature will peak at different wavelengths. The hotter the object, the shorter and bluer the wavelength.

## Star types

**Ae/Be Herbig**—More massive analog of T Tauri. A young star 2-8 M.

**Alpha Cygni**—Variable B or A supergiant w/ non-radial pulsations. Prototype is Deneb.

**AM Canum Venaticorum**—Rare type of cataclysmic variable, where WD accretes hydrogen-poor matter from donor star (helium WD, low-mass helium star, or evolved MS star).

**AM Herculis**—see "*polars*"

**Beta Cephei (Beta Canis Majoris)**—Class B stars with small rapid variations in brightness due to pulsations of star's surface. Pulsations driven by kappa mechanism and p-mode pulsations.

**BY Draconis**—K or M MS variables. Quasiperiodic variations due to rotation of star and star spots.

**Cataclysmic variable (CV)**--Stars with occasional violent outbursts due to thermonuclear processes. Many are close binary systems.

**Cepheid (anomalous)**--Pulsating variable on the instability strip, similar to RR Lyrae but with higher luminosities.

**Cepheid (classical)**--A pulsating Population I giants and supergiants whose luminosity and pulsation period are related.

**Cepheid (double-mode)**--Cepheid that pulsates in two modes at the same time, usually the fundamental and first overtone.

**Cepheid (type II)**--Population II Cepheids.

**Delta Cephei**--see "*classical cepheids*."

**Delta Scuti**--Also known as "*Dwarf Cepheids*." Class A stars on or near the main sequence at the lower end of the instability strip.

**DQ Herculis**--see "*intermediate polars*."

**Dwarf Nova**--Close binary system with a WD, red dwarf, and accretion disk. Variations are caused by instabilities in the disk.

**Eruption Variable**--Varies due to violent processes and flares in its chromosphere and coroneae.

**FK Comae Berenices**

**FS Canis Majoris**—Eruptive variable star. Likely in binary system w/ mass exchange. Consists of B MS star in dust envelope.

**FU Orionis**--A pre main sequence star that experiences dramatic change in magnitude and spectral type due to abrupt mass transfer from accretion disk onto star.

**Gamma Cassiopeiae**--*also known as "shell star."* An eruptive variable with a spectrum that suggests a circumstellar disk of gas. Irregular variations are due to outflow of matter.

**Gamma Doradus**--Stars that vary due to non-radial pulsations of their surface. Typically young F or late A main sequence stars.

**Irregular Variables**--Pulsating variables with little or no periodicity. Includes most red giants.

**Lambda Eridani**--Be stars with very regular variations due to either non-radial pulsations, inhomogeneous rotating discs, or the rotation of the star itself.

**Luminous Blue Variable**--*also known as "S Doradus."* Massive evolved stars that show unpredictable and sometimes dramatic variations in both their spectra and their brightness.

**Microquasar**--A smaller cousin of the quasar, sharing the same characteristics, strong and variable radio emission, often resolvable as a pair of radio jets, and an accretion disk surrounding a compact object which is either a black hole or a neutron star.

**Mira**--Pulsating red giants on AGB.

**Novae**--Close binary systems with an accreting WD and low-mass MS star.

**Polars**--Highly magnetic type of CV binary system, that produces polarized light. System contains accreting WD and low-mass donor star(usually red dwarf), with synchronous rotation (tidal locking).

**Polar (intermediate)**--Type of CV binary system where the accretion disk is disrupted by the WD's magnetic field. Infalling matter follow the magnetic field in accretion streams.

**PV Telescopii**--Hydrogen-deficient, helium supergiants.

**R Coronae Borealis**--Rare luminous, hydrogen-poor, carbon-rich supergiant that spend of the time at max brightness, fading at irregular intervals.

**Recurrent Novae**--Similar to novae, but with two or more smaller outbursts in recorded history.

**RR Lyrae**--Short period pulsating giants, usually class A. Older and less massive than Cepheids.

**RS Canum Venaticorum**--Close binary system with active chromospheres that cause large stellar spots.

**RV Tauri**--Yellow supergiants whose light variation alternates between deep and shallow.

**S Doradus**--*see "Luminous Blue Supergiant."*

**Semiregular variable**--Giants and supergiants with periodicity and intervals of irregular variation.

**SS Cygni**--*prototype for "dwarf nova."*

**SU Ursae Majoris**--Two distinct kinds of outbursts: one faint, frequent, and short; the other is bright, less frequent, and longer. Physically similar to U Gem stars.

**SW Sextantis**--A CV where the emission lines of hydrogen and helium are not doubled.

**SX Arietis**--B type main sequence variables that exhibits strong magnetic fields and intense He I and Si III spectral lines.

**SX Phoenicis**--Cousin to Delta Scuti, pulsating A-F type with low metallicity.

**Symbiotic Binary System**--Close binary with a red giant and a hot blue star, both embedded in nebulosity, that shows semi-periodic, nova-like outbursts.

**T Tauri**--young star <2 M.

**U Geminorum**--Suddenly brightens after intervals of quiescence at minimum light.

**Wolf-Rayet**--Evolved, massive stars that have completely lost their outer hydrogen and are fusing helium or heavier elements in the core. Spectrum indicates high surface temperature, surface enhancement of heavy elements, and strong stellar winds.

**W Ursae Majoris**--*also known as "low-mass contact binary."* Type of eclipsing binary variable star. These stars are close binaries of spectral types F, G, or K that share a common envelope of material and are thus in contact with one another.

**Z Andromedae**--*prototype for "symbiotic variable."*

**Z Camelopardalis**--Shows cyclic variations interrupted by intervals of constant brightness called "standstills." Physically similar to U Gem stars.