Question B: A Spectroscopic Eclipsing Binary System

Near alpha Aurigae is located a spectroscopic eclipsing binary system (star A and star B) which have nearly circular orbits about their common center of mass with a period of 88.2 days. The system is oriented such that the orbital plane is viewed edge-on as seen from the Earth. Hydrogen lines are observed in the spectra of both components. Data for three of these lines are given in the table below. (The units are nanometers.) The maximum and minimum observed values of the wavelengths are given for each star. From the Doppler shifts evident in the data, it is clear that the stars have different orbital speeds, hence different masses. From the data, determine:

a) **How fast and in which direction along the line of sight the entire system is moving relative to the Earth.**
b) **The orbital speed of each star and the distance between the two.**
c) **The mass of each star.**

	Star A		Star B	
Laboratory	**Minimum**	**Maximum**	**Minimum**	**Maximum**
656.273	**656.202**	**656.532**	**656.332**	**656.402**
486.133	**486.080**	**486.325**	**486.177**	**486.228**
434.047	**434.000**	**434.218**	**434.086**	**434.132**

SOLUTION

Useful Equations

$$\frac{\lambda - \lambda_\circ}{\lambda_\circ} = \frac{v_r}{c}$$ Doppler Shift

$$2\pi a = v\mathrm{P}$$

$$\left(M_A + M_B\right) = \frac{a^3}{\mathrm{P}^2}$$ Kepler's Third Law

Useful Definitions

Speed of Light = $3.0 \times 10^5 \, {}^{km}\!/\!{}_{s}$

1 AU = $1.5 \times 10^8 \, km$

Part A. Use the first equation and first row of H-line emission data to determine the min/max radial velocities of each star.

<u>StarA</u>

Min: $\frac{656.202nm - 656.273nm}{656.273nm} = \frac{v_r}{c}$ $\quad \Rightarrow v_r = -32.5 \, {}^{km}\!/\!_{s}$

Max: $\frac{656.532nm - 656.273nm}{656.273nm} = \frac{v_r}{c}$ $\quad \Rightarrow v_r = 118.4 \, {}^{km}\!/\!_{s}$

<u>StarB</u>

Min: $\frac{656.332nm - 656.273nm}{656.273nm} = \frac{v_r}{c}$ $\quad \Rightarrow v_r = 27.0 \, {}^{km}\!/\!_{s}$

Max: $\frac{656.402nm - 656.273nm}{656.273nm} = \frac{v_r}{c}$ $\quad \Rightarrow v_r = 59.0 \, {}^{km}\!/\!_{s}$

Since each star traverses a circular orbit, Kepler's Second Law requires that the velocity of each star around the center of mass is constant.

The average velocity of StarA = The average velocity of StarB
Since the stars maintain a constant distance relative to each other (remain in their circular orbits), the average velocity = The velocity of the system.

$$v_{system} = \tfrac{1}{2}\left(v_{r_{\min}} + v_{r_{\max}}\right) = \tfrac{1}{2}\left(-32.5 \, {}^{km}\!/\!_{s} + 118.4 \, {}^{km}\!/\!_{s}\right) = \tfrac{1}{2}\left(27.0 \, {}^{km}\!/\!_{s} + 59.0 \, {}^{km}\!/\!_{s}\right)$$

$v_{system} = 43.0 \, {}^{km}\!/\!_{s}$ away from the Earth.

Part B. Use the second equation to determine the radius of orbit.

Orbital velocity:

$v_{orbital} = v_{radial} + v_{system}$

<u>StarA</u>: $v_A = 75.4 \, {}^{km}\!/\!_{s}$

<u>StarB</u>: $v_B = 16.0 \, {}^{km}\!/\!_{s}$

Convert the units of Period (P) from days to seconds:

$$\text{P} = (88.2 days)\left(\frac{24hrs}{1day}\right)\left(\frac{3600\sec}{1hr}\right) = 7.6 \times 10^{6} \sec$$

$$\frac{2\pi r_{StarA}}{\mathrm{P}} = v_A$$

$$\frac{2\pi r_{StarB}}{\mathrm{P}} = v_B$$

$$\frac{2\pi a}{\mathrm{P}} = \frac{2\pi}{\mathrm{P}}\left(r_{StarA} + r_{StarB}\right) = \left(v_A + v_B\right)$$

$$2\pi a = \left(91.4\ {}^{km}\!/\!_{s}\right)\left(7.6\times10^{6}\, s\right)$$

$$\Rightarrow a = 1.1\times10^{8}\, km$$

Convert the units of Orbital Radius (a) from kilometers to AU.

$$a = \frac{\left(1.1\times10^{8}\, km\right)}{\left(1.5\times10^{8}\, km\right)} = 0.73AU$$

Part C. Use the third equation to determine the combined mass of the stars.

Convert the units of Period (P) from days to years:

$$\mathrm{P} = \left(88.2 days\right)\left(\frac{1 year}{365 days}\right) = 0.242 years$$

$$M_{StarA} + m_{StarB} = \frac{(0.73AU)^3}{(0.242yr)^2} = 6.64 M_{\odot}$$

$$\frac{M_{StarB}}{M_{StarA}} = \frac{v_{StarA}}{v_{StarB}} = \frac{118.4\ {}^{km}\!/\!_{s}}{59.0\ {}^{km}\!/\!_{s}} = 2.0$$

$$\Rightarrow 2M_{StarA} = M_{StarB}$$

$$M_{StarA} = 2.21 M_{\odot}$$

$$M_{StarB} = 4.42 M_{\odot}$$